

APPENDIX A: Lesson Plans

(Summer 2014 | RL 1)

Date: 25/05/2014 **Time:** 11:15

Aspect of mathematics: Introduction to algebra: missing number problem

<p>Accountable learning intentions and success criteria</p> <p>Describe what you want the class to be able to do by the end of the lesson and what that will look like.</p>	<p>CP1.....</p> <p>Success criterion for this lesson</p> <p>Will be able to solve problems with one unknown.</p>		<p>CP2.....</p> <p>Success criterion for this lesson</p> <p>Will be able to solve problems with one unknown, and with the = sign in an unfamiliar place</p>		<p>CP3.....</p> <p>Success criterion for this lesson</p> <p>Will be able to solve problems with unknowns on both side and with the introduction of a letter to represent an unknown.</p>	
<p>Stage of lesson sequence</p> <p>Describe key elements in the stages of your RL plan. (You can add more).</p>	<p>How you predict CP1 will respond</p>	<p><i>How they are observed to respond</i></p>	<p>How you predict CP2 will respond</p>	<p><i>How they are observed to respond</i></p>	<p>How you predict CP3 will respond</p>	<p><i>How they are observed to respond</i></p>
<p>Students will be introduced to Numicon with the instruction to find all the ways they can to make 10. This will be an assessment opportunity to see which children can find number bonds. They will then be asked if they can find a way to record their results.</p> <p>(25 minutes)</p>	<p>He will be able to find $3+7=10$. He will be reliant on the shape and colour. He will record his results as a sentence.</p>	<p>Excellent. He used number bonds to make 10. Good recording (he didn't use an = sign</p> <p>Eg. $6+4$, $5+5$, $10+0$)</p>	<p>He will be organised and maybe find the number bonds in order. He will need to be prompted to use 3 numbers to make 10. He will record his results as an equation.</p>	<p>Good. Using 4 numbers $4+1+2+3=10$. Always using $_{+}_{+}_{+}=10$ etc. with the 10 on the left hand side. This was consistent with everyone in the room.</p>	<p>He will be using more than 2 numbers to make 10 and will record his results accurately on the mini-whiteboard.</p>	<p>Started to work on number bonds straight away with $9+1$, but got distracted by another pupil and started to fill in the rest of the board. When asked he was able to explain correctly what was happening. He wrote sentences out using words instead of numbers</p>

<p>Demonstration. Teacher will introduce the word “balance” and the scales. She will take one of the students’ previous examples and use the scale to introduce the = sign. She will then turn the scales around so they can see that $8+2=10$ is the same as $10=8+2$. She will then demonstrate a missing number problem. The students will then have a variety of missing number problems stuck up around the classroom at various levels of difficulty for them to attempt.</p> <p>(25 minutes)</p>	<p>He will use trial and error to find the correct result. He will focus on 2 numbers less than 10.</p>	<p>He was really enjoying himself. He noticed the pattern with the number bonds and wanted to challenge himself</p>	<p>He will try one of the simpler ones and will use number bonds to do it. He will then be encouraged to try a more difficult one such as</p>	<p>With $6+3=_{\quad}+1$ he (and other pupils in the class thought that the missing number should be 9).</p>	<p>He will be given a more difficult one to do and the teacher will work with him. There will be a challenge activity if he finishes.</p>	<p>Was having a lot of trouble with $_{\quad}+3=3+_{\quad}$. He was trying to write $3+3=6$.</p>
<p>Teacher will put up a variety of completed problems on the board, some of which are incorrect. The students will be required to find the incorrect solutions and correct them on mini-whiteboards. Assessment opportunity.</p> <p>(10 minutes)</p>	<p>He will go for $1+3=9$</p>	<p>He looked at the numicon pieces in order to try and correct $3+1=10$. Got it wrong. He corrected this with a piece of numicon in his hand</p>	<p>He will go for $7=3+6$</p>	<p>Immediately corrected the first one. Did the other easier one. With prompting was able to correct the trickier one.</p>	<p>He will go for $6+3=7+1$</p>	<p>He managed the trickiest one with prompting.</p>

(Summer 2014 | RL 2)

Date: 6/06/2014

Time: 11:00

Aspect of mathematics:

The meaning of the equals sign (algebra)

<p>Accountable learning intentions and success criteria</p> <p>Describe what you want the class to be able to do by the end of the lesson and what that will look like.</p> <p>For each case pupil state what you hope this pupil will be able to do by the end of this lesson that is new/progress in the adjacent boxes</p>	<p>CP1.....</p> <p>Success criterion for this lesson</p> <p>To understand the equals sign as a symbol of equivalence and solve simple equations using symbols other than an empty box.</p>	<p>CP2....</p> <p>Success criterion for this lesson</p> <p>To understand the equals sign as a symbol of equivalence and solve more complex equations using symbols other than an empty box.</p>	<p>CP3....</p> <p>Success criterion for this lesson</p> <p>To understand the equals sign as a symbol of equivalence and solve simple equations (possibly using multiplication) using symbols other than an empty box.</p>
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Stage of lesson sequence Describe key elements in the stages of your RL plan. (You can add more).	How you predict CP1 will respond	<i>How they are observed to respond</i>	How you predict CP2 will respond	<i>How they are observed to respond</i>	How you predict CP3 will respond	<i>How they are observed to respond</i>
Using the equivalence of money to reinforce the meaning of the equals sign as a symbol of equivalence, rather than a symbol which means “calculate”. After a demonstration on the board, the children will be have a cardboard = sign and will be required to come up with different combination of coins and notes which make the “money equations” true. (25 minutes)	He will come up with equations using single coins on one side and perhaps £2 coins on the other.	During the starter, CP1 made correct combinations using £1 and £2 coins	Will experiment with using notes and different combinations of coins and notes.	CP2 did this successfully and recorded his work well. See photos.	As CP2.	CP3 did make these more complex combinations and recorded the results accurately
Students will be required to balanced number sentences using symbols standing in for the missing numbers. As in the last lesson, after a demonstration there will be differentiated questions posted on the walls of the classrooms. (20 minutes)	Will attempt to answer questions such as $3+\Delta=5$	CP1 was using the Numicon (he asked to use it) tackling green problems like the one we predicted. He is gaining in confidence with this.	Will attempt to answer questions such as $4+\Delta=6+3$	Started with a green $7-?+1$ and got it straight away. Partner did $3+5=8+3$. CP2 does a few more greens and then successfully attempts the one above.	As CP2, but also may attempt $3x\Delta=12$. May also begin to use letters instead of symbols.	Started with $10=\Delta+3$ and got it correct. Then $4+5=\Delta+3$ no problems. Does all of the blue and also the cream – no probs. I give extension $3xx-6=9$. CP3 and the girl next to him complete this using trial and error (check-and-guess)
Students will be given differentiated questions and will work first of all on their own, and then in groups in order to solve four equations which will then be used to crack a code. (10 minutes)	As above, but will work independently and with no resources.	Big problems with minus problems as per CP2. All very enthusiastic!	As CP1.	Very good with two of them but a great deal of difficulty with $0-3=12-7$	As CP1.	Gets all three straight away no probs

What were they able to do? (What progress have they made and how do you know?)		During the starter, CP1 was able to notice the missing operations from the number sentence		Need to be given the tools to deal with negatives.		Needed to be extended.
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(Summer 2014 | RL 3)

Date: 13/06/14 Time: 11:00 Aspect of mathematics: Algebra (function machines)

Accountable learning intentions and success criteria Use function machines to begin to understand “undoing” or the inverse	CP1..... Success criterion for this lesson Be able to use a function machine to “undo” simple functions.		CP2..... Success criterion for this lesson As per CP1 but with more confidence		CP3..... Success criterion for this lesson Be able to use function machines to find the missing number in functions with more than one step	
Stage of lesson sequence	How you predict CP1 will respond	<i>How they are observed to respond</i>	How you predict CP2 will respond	<i>How they are observed to respond</i>	How you predict CP3 will respond	<i>How they are observed to respond</i>
Recap on previous lesson’s work with each child trying one of the missing number problems around the room. There will be a couple of extra challenging ones. (5 minutes)	He will remember how to work out simple equations (eg $?+3=5$)	CP1 happy to show what he remembered, able to $5=?+1$ correctly but still needed support and encouragement to attempt at the next level.	Will pick a blue one eg. $5+?=2+9$	Completed all levels and went on to attempt the new level of challenges eg. $4+?=6 \times 4$ Wanted to take them home to complete.	He will try one of the new challenging tasks. eg. $? \times 2=6+4$	Able to do the new challenging tasks with a few errors but was correcting.

<p>There will be a demonstration by the teacher on function machines. What does this machine do? If I put a 2 in what will come out? If a 5 comes out what have I put in?</p> <p>The children will then try a series of different activities based on this:</p> <p>Lower – working with the TA using manipulatives demonstrating the machine.</p> <p>Middle – a series of levelled questions to see if they can work independently</p> <p>Higher – working with the teacher leading to more complex functions which need “undoing”</p> <p>(25 minutes)</p>	<p>He will be able to identify an operation when using resources, and then use the inverse.</p>	<p>Able to say what the answer would be through the function machine, it will turn to a 3.</p> <p>With cubes could take away to show the inverse.</p> <p>Drawing circles and number sentences</p>	<p>He will be using the inverse for single operation problems.</p>	<p>Some confusion with inverse of x as $-$ but could correct after explanation with adult.</p> <p>After some additional support about recording the inverse operation could complete all of them include higher numbers including</p> <p>$? + 15 = 74$</p>	<p>He will use the inverse to work out the input. He will perhaps work up to solving 2-step functions</p>	<p>Made an error initially during teaching $? + 4 = 6$ he said it is 10 but after teaching told me ‘No, I think its 2’</p> <p>Then he completed two step with teacher.</p>
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APPENDIX B: Post Lesson Discussion Records

Post-Lesson Discussion: Research Lesson 1

	CP1	CP2	CP3
<p>What progress did each pupil make? Was this enough?</p> <p>What about others in the group of learners they typify?</p> <p>Do we need to revise our assessment of any pupils?</p>	<p>Confident with number bonds to 10. Reliant on shape; recorded on whiteboard with prompting 7 + 3, 8 + 2 etc. Did as expected –</p>	<p>Started translating numicon into number sentences straight away. Struggled when asked to order number sentence differently (10 = ____ + 4)</p>	<p>Went straight to number bonds to 10 reflecting his comfort zone. “ 5 then 3 then 2 is a block of 10...”</p>
<p>How did the teaching being developed help or hinder the pupils’ learning? (Maybe a bit of both)</p> <p>What surprises were there?</p> <p>Did we find out anything of note about the way they were learning?</p>	<p>Use of Numicon developed his understanding of balancing number sentence. “ Both sides need to be the same...?”</p>	<p>Teaching using scales helped CP2 understand balancing of sides. Could use scales to solve:</p> <p>2 + 4 = 1 + ____</p> <p>Conventional number sentence format ingrained. Struggled with turning the number sentence around. Needed lots of prompting with examples such as:</p> <p>10 = 8 + 2</p>	<p>CP3 could correct mistakes using the balancing scales but needed prompting and relied upon trial and error.</p> <p>This helped him understand balancing of sides. The plenary consolidated his understanding of this and he could spot mistakes and correct answers on board.</p> <p>3 + <u>1</u> = 10 – could spot mistake and correct independently.</p>
<p>What aspect(s) of our teaching could be adjusted next time to improve the progress of our case pupils and all pupils</p>	<ul style="list-style-type: none"> • Consolidation of the meaning of equals sign • More play with this support group • Consolidation of ‘turning a number sentence around’ 		
<p>So what should we try next time?</p>	<ul style="list-style-type: none"> • Challenges involving shapes / letters and missing numbers using images of Numicon on scales • Teaching of strategies for balancing and finding missing numbers without resource 		

Post-Lesson Discussion: Research Lesson 2

	CP1	CP2	CP3
<p>What progress did each pupil make? Was this enough?</p> <p>What about others in the group of learners they typify?</p> <p>Do we need to revise our assessment of any pupils?</p>	<p>CP1 used the Numicon again to solve the problems. Progressed to not using the resource in the plenary. In the post lesson interview, he said that ‘shapes can stand for numbers’.</p> <p>Made expected progress with reliance on adult support; others in group made similar progress.</p>	<p>Now very confident with balancing problems where there were operations on both sides of the equals sign – could not do this independently last lesson.</p> <p>Others in group need to be moved to support group (Q and B) as could not manage similar problems without support or resource.</p>	<p>Exceeded expectations; very confident with all that was put in front of him.</p> <p>Could have worked on more challenging problems. Others in group could also have benefited from such challenges.</p>
<p>How did the teaching being developed help or hinder the pupils’ learning? (Maybe a bit of both)</p> <p>What surprises were there?</p> <p>Did we find out anything of note about the way they were learning?</p>	<p>The progression into the context of money and into code breaking developed CP1’s understanding of the type of problems.</p> <p>Relied confidently on resource (Numicon – which he obtained voluntarily) where others in group relied upon trial and error. They are ready for more strategies (e.g. simple inverse – function machines)</p> <p>When triangles were used, at least 2 children said ‘the number must be 3 because it has 3 corners/sides’.</p>	<p>Familiar context of money was very useful for CP2. He could create balanced equations and this helped the development of his learning when moving on to the algebra problems. The slower pace and the increased opportunities for independent activities in an exciting context (money, code cracking, sealed envelopes) helped CP2’s motivation, excitement for the learning and understanding. He worked quickly and confidently.</p> <p>He and his group felt in charge, picked appropriate problems and produced good outcomes.</p>	<p>CP3’s learning was helped with the slow, clear progression of the teaching.</p> <p>He exceeded expectations (along with his group) and could have benefited from extension problems.</p> <p>He wanted more challenges and articulated this in his interview.</p>
<p>What aspect(s) of our teaching could be adjusted next time to improve the progress of our case pupils and all pupils</p>	<ul style="list-style-type: none"> • Inverse strategies to solve algebra problems • Extra challenges for more able (multiplication/division and addition/subtraction on either side of equals sign?) 		

So what should we try next time?	<ul style="list-style-type: none"> • Use of function machines • Stay with the slower but snappy lesson style with clear contextualised progression • Use of letters instead of shapes • Challenge the extension group
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Post-Lesson Discussion – Research Lesson 3

	CP1	CP2	CP3
<p>What progress did each pupil make? Was this enough?</p> <p>What about others in the group of learners they typify?</p> <p>Do we need to revise our assessment of any pupils?</p>	<p>At the end of the lesson CP1 had an understanding of inverse operations; e.g. inverse of addition is subtraction. He could solve problems using inverse with support.</p> <p>Others in group slightly exceeded this, as they could solve some problems without support.</p>	<p>CP2 used relied upon trial and error at first to solve the problems. However in the plenary, he could talk though a 'I think of a number' problem working backwards and using the inverse operation. He exceeded expectations in the sense that he could inverse multiplication when put in the context of doubling and halving.</p> <p>Others in his group typified this progress and some exceeded.</p>	<p>CP3 could solve two step inverse problems working backwards, and set out his calculations systematically and repeated success in a number of examples. He kept asking for more challenging problems.</p> <p>This was typified ion the group and some learners from the core group moved in to the extension group as they required extended. The fluidity between groups was very successful using AFL to move children on when required.</p>

<p>How did the teaching being developed help or hinder the pupils' learning? (Maybe a bit of both)</p> <p>What surprises were there?</p> <p>Did we find out anything of note about the way they were learning?</p>	<p>We were very surprised at the level in which all learners could solve problems from previous lessons which were on the walls (they had remained there for a week). They could balance the equations and find the 'disguised' numbers with ease.</p> <p>Adult support who were properly briefed of what we wanted them to achieve at the end of lesson (and the progressive pace expected) helped develop understanding of inverse operations.</p>	<p>The importance of pace, breaking all learning down was very important for all core and support learners in this class.</p> <p>Children having control over the difficulty of the problems they tackled increased motivation and they all chose problems matched to their ability.</p>	<p>Teacher working with group had huge impact on stretching the extension group. This focussed support allowed the learners to have misconceptions cleared very early. This support enabled the children to understand the backward process and undoing of multi-step problems.</p> <p>Some children, when recording their calculations, used the equals sign inappropriately – it was used to signify next steps rather than its meaning of 'equals'. This was addressed in some cases, but not all.</p>
<p>What aspect(s) of our teaching could be adjusted next time to improve the progress of our case pupils and all pupils</p>	<ul style="list-style-type: none"> • Visual cue for reversal of functions in multi-step problems to assist in understanding the order of which to perform operations (backwards!) 		
<p>So what should we try next time?</p>	<ul style="list-style-type: none"> • Use of larger numbers where written methods are required, hence using number sentences they might not immediately recognise (e.g. ? + 42 + 112) 		