Research lesson accounts

The class we would be focusing on was a year 4 class consisting of 27 boys and girls who had been put into attainment-based sets, having until recently been taught in mixed-ability classes. There was some concern from the class teacher [from now referred to as Research Teacher 3, or RT3] that there was a general feeling of low self-esteem within the class, and that this would be something else to explore during the Research Lessons. Together we selected three case study pupils that we would focus on, all with varying abilities and needs, so that we could think about how our teaching affected different pupils.

Case Pupil 1 [henceforth CP1] was currently attaining a maths level 2. He found recall of all mathematics knowledge very difficult and found number bonds up to 10 challenging. He was reliant on resources such as counting on fingers and found focus and concentration challenging over time. He had huge reliance on adult support during “independent” activities. He has occupational therapy support for gross and fine motor skills.

Case Pupil 2 [CP2] has possible dyslexia (but this has not been tested) and his reading was putting him at a disadvantage (he is level 1 in reading), though he is stronger in mathematics. Practising and rehearsing made him more successful and confidence was a big issue with him. We wanted to become clearer about what his barriers to learning were.

Case Pupil 3 [CP3] appeared to have significant gaps in his maths knowledge. He grasped some concepts quickly whilst with others he would need rehearsal and prompting. When he had prior knowledge or adult support he could achieve well. He had attended a Key Stage 1 nurture group and had some low-level behaviour issues, although not so far in the new maths groups.

After identifying the case pupils we decided on our focus for the first lesson. The class teacher [RT3] was to teach the lesson whilst I [RT1] and the other year 4 primary teacher [referred to as RT2] would observe the learning of primarily the case pupils, but also of the other children in the class.

Research Lesson 1

We decided to focus on missing number problems and the meaning of the equals sign for this introductory lesson. We wanted the children to be able to solve equations with one unknown by really understanding that the equals sign meant “is equivalent to” instead of “find the answer”, a common misconception detailed in my research above. We also wanted to see what effect the use of manipulatives would have on the learning of the pupils, and to this end we decided to use the Numicon resource, also outlined above. Most of the students would be unfamiliar with this resource at the beginning of the lesson. We had three outcomes that we thought the CS Pupils, and other pupils at their levels, could achieve during this lesson.

- We wanted CP1 to be able to solve problems with one unknown ie. in the form 3 +?= 10
- We wanted CP2 to be able to solve problems with one unknown, and with the equals sign in an unfamiliar place eg. 9 =? + 3
- We thought that CP3 would be able to solve problems with unknowns on both sides and with the introduction of a letter to represent an unknown (something unfamiliar to all pupils at this stage) eg. ?+5=2+?

We first wanted the children to become familiar with the resource, so the lesson was introduced with all of the children closing their eyes and being given a “10” piece.

CP1 called the piece a “10” before anyone else, suggesting familiarity with the resource. When asked how they knew it was a 10, the pupils in the class showed a wide range of mathematical knowledge, comments including:

“I counted up in twos and there’s five twos and that makes 10”

“It’s an array”

“I put my fingers in the hole and counted up to 10”

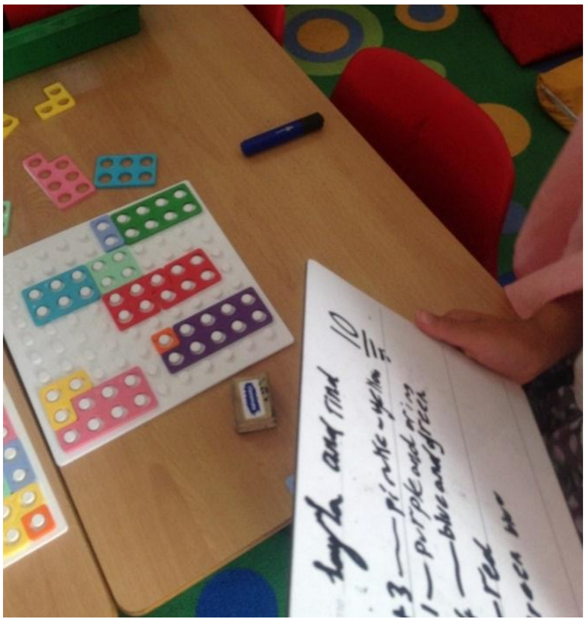
“I saw that it was 5 on one side and 2 on the other and I know that made 10”

This shows how an open question can provide an assessment opportunity and gives pupils the chance to use their imagination and make links with other areas of mathematics.

After this first activity the children were asked to find all of the ways that they could make 10 with the Numicon pieces and then to think of a way to record their results.

We thought that CP1 would manage this activity and stick with the number bonds that he was familiar with. We thought that he would be reliant on shape and colour and that he would record his results as a “number sentence” (the primary school staff used this phrase in preference to the mathematical word “equation”. CP1 managed this activity very well. He used number bonds to make 10 and recorded his answers, although he didn’t use an equals sign, preferring to just list the numbers with a + in-between.

We thought that CP2 would be very organised and perhaps find the number bonds in order ie. 1+9=10, 2+8=10 etc. We thought that he would need to be prompted to use three numbers and we thought that he would record his results as an equation. He approached this task well. He was using four number such as 4+1+2+3=10, using the equals sign accurately. Interestingly, he always put the equals sign on the right hand side, at the end of the number sentence, which was consistent with everyone in the class. However, this is not entirely surprising, as the task was to “make a number”, but perhaps this goes some way to showing that the pupils saw the equals sign as meaning, “the number that is made is…”



Using Numicon to make number bonds to 10

We thought that CP3 would find the task easy and be able to record his results accurately on a whiteboard. He started to work on number bonds straight away, but soon got distracted by another pupil and started to fill in the rest of the workboard in a haphazard way. When asked, he was able to explain correctly what was happening. He wrote out sentences in words rather than numbers. These observations suggested to me that he understood mathematical concepts but lacked formality in the way he approached his working.

During the next part of the lesson we wanted to emphasise the meaning of the equals sign. To this end we used a balance scale with an equals sign in the middle to try and get the children to see that one side of an equation needed to be equivalent to the other side. The teacher started by taking one of the children's previous examples and used the scale to introduce the equals sign, taking a piece of card with an equals in it and putting it in the middle of the scales. Numicon pieces were then put in the boxes on each side of the scale and the children could see that the scale was balanced. She then turned the scales around to show that 8+2=10 was the same as 10=8+2. When demonstrating many pupils had great difficulty with this and during the post-lesson discussion we were trying to understand why this might be. We thought it might be because the children were unable to see what was in the scales and didn't have the spatial awareness to remember when the scales were turned around. The teacher then demonstrated a few missing number problems. When students suggested answers to the problems the teacher tried it out by putting Numicon pieces in the scales and seeing if they balanced.

The students then had a variety of missing number problems pinned up around the classroom in different colours depending on their difficulty for them to attempt. They had scales as resources if they wished to use them.

We thought that CP1 would use trial and error to find the correct result and that he would focus on two numbers less than 10. He was really enRT3ying himself. He noticed the relationship between number bonds and what he was working on and he really wanted to challenge himself. During his post-lesson interview, CP1 said that using the scales had been his favourite part of the lesson. He also used an interesting turn of phrase when describing the scales. He said,

"You've got to see if it's like a mirror"

Something interesting happened with CP2 and other pupils in the class. He was focusing on the problem 6+3=?+1 and he thought the missing number would be 9. This was happening all over the classroom and supports the idea that many pupils see the equals sign as “what’s the answer” rather than a sign of equivalence. When prompted, and by using the scales he was able to correct himself. During the post-lesson interview CP2 said that he thought the green problems were easy (problems such as 3+?=9) and that the blue problems (like the one I described above) were difficult. We thought that CP2 needed more time and another way of seeing the meaning of the equals sign.

CP3 was working on more difficult equations. With the help of others on his table, and the teacher he was able to do questions such as 6+3=?+1. He started off with the same difficulties as CP2, in that he wanted to write 9 as the answer. He eventually overcame these difficulties and was able to answer a similar question correctly in the post-lesson discussion. Everyone working at a higher level was having difficulty with ?+3=3+?. CP3 was trying to write out 3+3=6 in answer to this problem. It was obvious that the whole class would need some more work on the meaning of operations and the equals sign.

For the final part of the lesson and as an assessment opportunity, the class teacher put some completed problems on the board, some of which were incorrect. The children were required to find the incorrect solutions and correct them on their mini-whiteboards. CP1 tried to correct 3+1=10. He looked at some Numicon pieces on the table next to him to try and correct it. Initially he still got it wrong, but he corrected this when I prompted him to take a piece of the Numicon in his hand. During his interview he was sheepish about having used the Numicon, thinking that he’d “cheated”, but it showed us that currently the use of manipulatives was vital for CP1. CP2 immediately corrected the first mistake and then corrected the other easier one. With prompting he was able to correct the trickier one. CP3 managed the hardest question without any prompting.

After the lesson we immediately interviewed all three case pupils. All three had enRT3yed the lesson and had all noted that the number sentences looked a bit different. CP1 said that,

“The number sentences were a bit different because there was one sum on one side and one sum on the other side. I know that they both equal the same number.”

CP2 said that, *“I need to practise when the equals switches around”* whereas CP3 said “*well they were different because normally you’d have 1 + 6 = 7. They were 10 equals then you have to write what it is”.*

We agreed during our post-lesson discussion that teaching using the scales had helped the children understand the balancing of sides. The children corrected mistakes using the scales and the “spot the error” consolidated understanding. For CP1 the use of Numicon developed his understanding of balancing number sentences with his comment, “both sides need to be the same”.

We decided that during the next lesson we needed to consolidate the meaning of the equals sign and “turning a number sentence around”, and that we needed more play with this support group. During the next lesson we would use challenges using shapes and letters and perhaps have missing number problems using images of Numicon pieces on scales. We would think about using strategies for teaching balancing and finding missing numbers without the physical resources for some pupils.

Research Lesson 2

We knew from Research Lesson 1 that the children were still having difficulty with the “equivalence” meaning of the equals sign, and wanted to explore this more in Research Lesson 2. We also wanted to introduce symbols other than an empty box to represent the unknown, so that children could be gradually introduced to the idea that symbols (including letters) could represent any number. The success criteria for this lesson were therefore set as follows:-

- CP1 was to understand the equals sign as a symbol of equivalence and solve simple equations using symbols other than an empty box
- CP2 was to understand the equals sign as a symbol of equivalence and solve more complex equations using symbols other than an empty box.
- CP3 was to understand the equals sign as a symbol of equivalence and solve simple equations (possibly using multiplication) using symbols other than an empty box.

One of the problems the children encountered with RL1 was that they saw a calculation with numbers on one side and immediately did the operation without considering the equivalence of the other side of the equation. We decided to use the equivalence of money to reinforce the meaning of the equals sign as a symbol of equivalence, rather than a symbol which means "calculate". After a demonstration on the board, the children would have a cardboard "=" sign and be required to come up with different combinations of coins and notes which made the "money equations" true.

We thought that CP1 would come up with equations using single coins on one side and perhaps £2 coins on the other, and this is what he did during the starter. He made correct combinations using £1 and £2 coins, understanding that both sides of the equals sign needed to be equivalent.

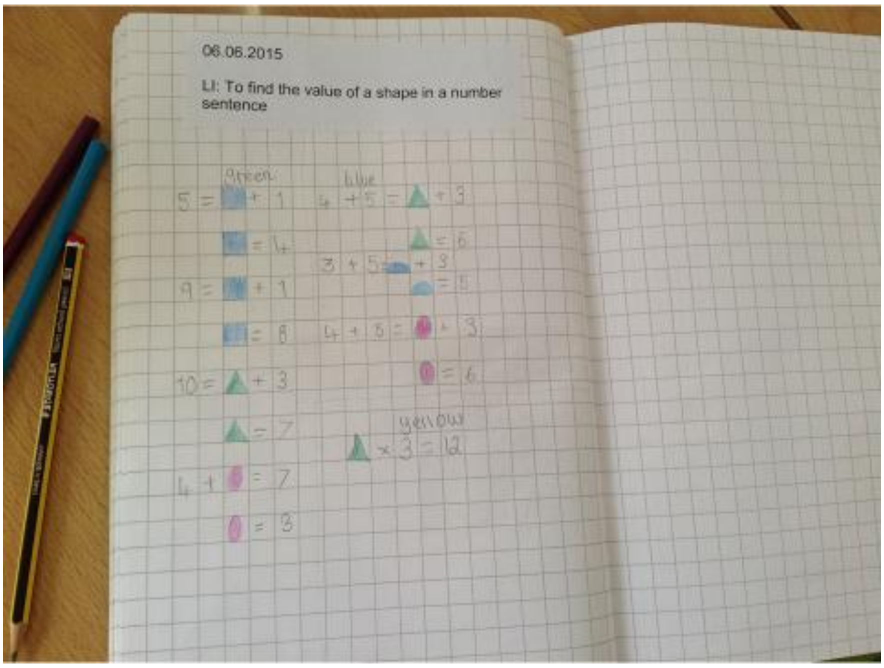
CP2 experimented successfully with using coins, and different examples of coins and notes. He recorded his work well, by drawing the coins, and writing the combinations in figures. We predicted the same outcome for CP3 and he did make these more complex combinations and recorded the results accurately.

Child using the equals sign to mean "is equivalent to"

After this starting activity, we decided that the students would be required to balance number sentences in different levels of difficulty, with the questions posted around the room as per RL1. The only difference from the first research lesson was that this time shapes were put in place of the empty boxes to represent "missing" numbers. It was interesting to note during the teacher demonstration that one pupil thought that the triangle must represent the number three because, *"it has three corners"*.

This time CP1 asked to use the Numicon to tackle some of the easier problems. We noted that he was gaining confidence with these problems. CP2 started with a green problem 7=?+1 and got it straight away, whilst his partner tried 3+5=8+3 , making the same error as the previous lesson. CP2 tried a few more green [easier] questions and then tackled the above question successfully.

We underestimated the ability of CP3 when devising the problems. We thought that based on his performance in the previous research lesson, he would be around the same point in his learning as CP2, but he started with 10=Δ+3 and got it correct. He then tackled 4+5=Δ+3 with no problems. He did all of the blue [medium] and also the cream [hard] with no problems. I then gave him an extension 3*x*-6=9. CP3 and the girl next to him completed this using trial and error (check -and - guess). This performance gave us the impression that what CP3 needed was the chance to rehearse, and to fill the gaps in his knowledge, as he was quick to grasp a concept which he'd been introduced to in the previous week, and had had difficulty with in the previous week. The higher attainers on this table had immediately jumped to using trial and error as a means to solving equations as they hadn't been taught any other ways of tackling this. We wondered whether it would be worth starting to tackle using the inverse at this early stage, to create the beginnings of another tool that the children could use.



Missing number problems

To finish the session, and as another assessment opportunity for the teacher the students were then given differentiated questions on which they needed to work individually, the answer to which would crack a code which would release a football for early play. All groups worked on this enthusiastically, although there were great problems with working with problems involving negative numbers with CP1 and CP2 eg. O-3=12-7. This was another instance where, without the understanding of using the inverse, they didn't have the correct tools at their disposal. Case pupil 3 managed all three problems straight away with no issues.

In the post-lesson interviews CP1 said that, *"I learnt a lot of things…. shapes can be numbers ",* and said that using Numicon really helped him:

"It helps me with the counting. On one of these questions it had 3 on one side and then it had a five on one side and then you put the 3 on and then the 2 onto the five and then you find out what it is"

When asked what he learnt, CP2 said the following:

RT2: "What did you learn?"

CP2: "I learned to find better for bigger numbers and longer ones."

RT2: "Can you remember what the learning objective was? What were we trying to do?"

CP2: "No. I have a short memory. We were trying to find out what numbers were behind the shape."

RT2: "Could you do that before?"

CP2: "A bit. Because if I do shapes with random numbers behind it, but since I got better at it I can do it"

RT2: "Which part of the teaching worked best for you?"

CP2: "The games. Especially code breaking, I liked that bit. And the money bit."

RT2: "Why do you think the money helped you?"

CP2: "Because I've seen it so it's easier"

RT2: "And why do you think the codebreaking helped you?"

CP2: "Because when I went to my friend's house they were playing batman gotham and they were solving a code-cracking. I got it with my first try."

This conversation shows how putting unfamiliar mathematical concepts in a familiar setting helped CP2 to access the work, and made him more confident and comfortable.

We asked CP3 what he'd learnt and he said, *"How to do problems with times tables. How to do ones with shapes. I've never done that before. The ones we did before we used squares. Today we used letters."*

However, when asked what could be changed about the lesson he said, *"Make it harder. I wasn't challenged enough. He* [the teacher] *could use division and make the number sentences longer. I like to be challenged because I'm really good at maths and when it's a challenge it's harder and I enjoy it more."*

When discussing the lesson, all three research teachers agreed that the case pupils had all made expected progress, with CP3 exceeding expectations. Others in the class of the same ability as CP3 could also have benefitted from an extra challenge.

The progression into the context of money and into code breaking developed CP1's understanding of the problems. He relied confidently on the resource (Numicon – which he obtained voluntarily) where others in group relied upon trial and error. We thought that they were ready for more strategies such as a simple inverse involving function machines. When triangles were used to represent the missing number, at least 2 children said 'the number must be 3 because it has 3 corners/sides'.

Familiar context of money was very useful for CP2. He could create balanced equations and this helped the development of his learning when moving on to the algebra problems. The slower pace and the increased opportunities for independent activities in an exciting context (money, code cracking, and sealed envelopes) helped CP2's motivation, excitement for the learning and understanding. He worked quickly and confidently. He and his group felt in charge, picked appropriate problems and produced good outcomes.

From our discussion, we decided that the next step was to introduce inverse strategies to solve algebra problems, to introduce extra challenges for the most able, and yet to stay with the slower but snappy lesson style with clear contextualised progression. We also wanted to introduce letters instead of shapes, to build familiarity at this early stage.

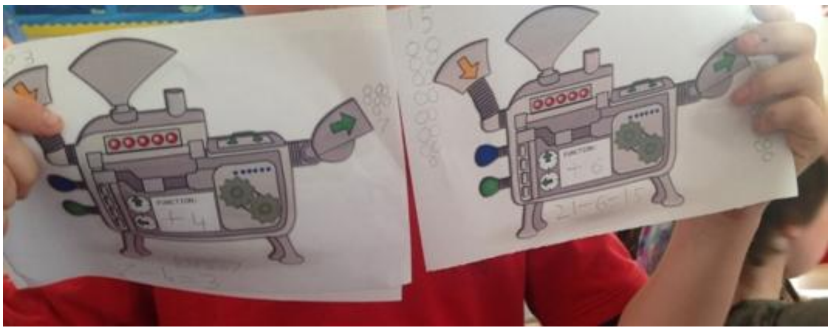
Research Lesson 3

From the research that I conducted it had been found that students encountering algebra in secondary school were still using guess-and-check methods to solve equations, and it had also been suggested that algebraic concepts be introduced gradually. We thought it would be useful to introduce the idea of the inverse and a function machine at this early stage to introduce a variety of tools to lower-attaining pupils who would not usually be given the opportunity with these sort of concepts. The success criteria were set as follows:

- CP1 would be able to use a function machine to "undo" simple functions.
- CP2 would be expected to achieve the same as CP2 but with more confidence.
- CP3 would be able to use function machines to find the missing number in functions with more than one step.

We decided to recap on the previous lesson's work with each child trying one of the missing number problems around the room. A couple of extra challenging ones had been added. The class teacher explained that having left the problems up over the course of three weeks had meant that the children were continually referring to them and explaining them to children in other maths sets. This meant that when they came back to them this week, they had not been forgotten and ensured some continuity.

For this assessment activity we expected that CP1 would still remember how to solve simple equations eg. ?+3=5. He was happy to show what he remembered and was able to solve 5=?+1 correctly. However, he still needed support and encouragement to attempt questions at the next level. We expected that CP2 would pick a medium level question to solve. In fact, he completed all levels and went on to attempt the new level of challenge eg. 4+?=6x4. He was very keen after this success to take them home to complete. CP3 was able to do the new challenging tasks with a few errors which he was able to self-correct. This encouraged us that the pupils now had a very good understanding of the meaning of the equals sign, with perhaps only a few of the lower-attainers still having difficulty.



The next part of the lesson consisted of a demonstration by the teacher of function machines asking: "What does this machine do?"; "If a put a 2 in what will come out?"; "If a 5 comes out what have I put in", until the pupils had a good understanding of what was meant by a function machine. After this, and based on pupil assessment over the previous two lessons, the children were grouped to work on different tasks as follows:

- Lower-attaining pupils including CP1 would be working with the Teaching Assistant using manipulatives (blocks and a picture of the function machine) demonstrating the machine and solving problems.
- Middle-attaining pupils including CP2 would be working independently on a series of levelled questions.
- Higher-attaining pupils including CP3 would be working with the teacher leading to more complex functions which needed "undoing".

We expected that CP1 would be able to identify an operation when using the resources and then use the inverse. When observing CP1, we could see that he was able to say what the answer would be when putting a number through the function machine, and using the cubes he was able to take away to show the inverse. He was recording his results by drawing circles and number sentences.

We anticipated some difficulty with the middle set of pupils as they were expected to work independently and indeed there was some confusion when using the inverse. CP2 could correct himself after an explanation from an adult. After some additional support about recording the inverse operation he could complete all of them including higher numbers such as ?+15=74. Despite this, we still had some reservations about the level of understanding (relational vs instrumental) regarding these two groups of pupils.

CP3 initially made an error during teaching. When answering ?+4=6 he said 10 (the problem we saw again and again in RL1 and RL2) but after teaching told RT2, "No, I think it's 2". He then completed two-step equations with the teacher.

To finish the lesson the pupils were to become "mind-readers" with some "I'm thinking of a number" problems. CP1 and pupils who were having difficulty went with the TA to answer the inverse of simple functions. CP1 was able to say that +4 was the inverse of -4 but found it difficult to keep track when explaining with the input and output numbers. He still needed to be supported and use manipulatives.

CP2 and CP3 tried some more challenging problems in the other group. They both, and others in the class, completed the more challenging work independently and talked in the plenary about solving a multi-step problem involving multiplication systematically.

After the lesson we interviewed CP1. When shown a function machine (15+6=21) and asked how you would find the number of counters which went into the machine, he said, *"you could halve it to 15",* but managed with prompting to correct himself to "*taking away*". He didn't really seem to understand what the word "inverse" meant. He liked the function machine on the Smartboard "*when it moved and made noises*" because it was "*fun*". He mentioned that in the Guess My Number section at the end "*everybody was holding up their boards and then their boards were the right answers and then I'd copy them"*, which suggests that he was still a little uncomfortable with the concept, or at least with the speed in which he was expected to answer.

In his interview CP2 showed a sound understanding of the word "inverse", as demonstrated in the following exchange:

RT1: "If I had a number sentence which was something plus 3 equals 8, how would you use the inverse to find out what that something was"
CP2: "You would do 8 take away 3 equals… 8, 7, 6, 5 and you would check it twice!"

He also showed his enjoyment at being able to tackle the most difficult problems by saying, *CP2: "I liked the bit where you had to do one star two star, and I almost got to the 3 star … because it was hard for me and I like challenges, cos I'm a top maths learner … but I'm a bit shy"*

CP3 similarly demonstrated a similar enRT3yment by mentioning a part of the lesson where, "*I thought it was going to be hard but it was actually easy*".

When asked what he had learnt that lesson he said, *"inverse … before I didn't actually know you could do the inverse … we were turning sums around"*. When asked, he managed to do a quite complex sum during the interview, and when asked what could be done to improve the lesson he said, *"make it harder … do harder sums".*

After this final research lesson we discussed the progress and surprises from each of the Case Study Pupils.

At the end of the lesson CP1 had an understanding of inverse operations; e.g. inverse of addition is subtraction. He could solve problems using inverse with support. Others in his group slightly exceeded this, as they could solve some problems without support. We were very surprised at the way in which all learners could solve problems from previous lessons which were on the walls (they had remained there for a week). They could balance the equations and find the 'disguised' numbers with ease. Adult support who were properly briefed of what we wanted them to achieve at the end of lesson (and the progressive pace expected) helped develop understanding of inverse operations.

CP2 relied upon trial and error at first to solve the problems. However in the plenary, he could talk though an 'I think of a number' problem working backwards and using the inverse operation. He exceeded expectations in the sense that he could inverse multiplication when put in the context of doubling and halving. Others in his group typified this progress and some exceeded. The importance of pace and breaking all learning down was very important for all core and support learners in this class. Children having control over the difficulty of the problems they tackled increased their motivation and they all chose problems matched to their ability.

CP3 could solve two step inverse problems working backwards, and set out his calculations systematically and with repeated success in a number of examples. He kept asking for more challenging problems. This was typified in the group and even some learners from the core group were moved into the extension group as they required extension work. The fluidity between groups was very successful, using "Assessment for Learning" to move children on when required. The fact that the teacher was working with the extension group had huge impact on stretching the most able. This focussed support allowed the learners to have misconceptions cleared up very early on and also enabled the children to understand the reverse/inverse process and undoing of multi-step problems. Some children, when recording their calculations, used the equals sign inappropriately – it was used to signify next steps rather than its meaning of 'equals'. This was addressed in some cases, but not all.